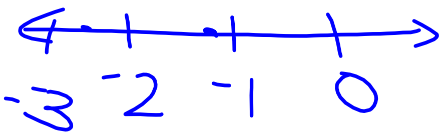
# The Greatest Integer Function

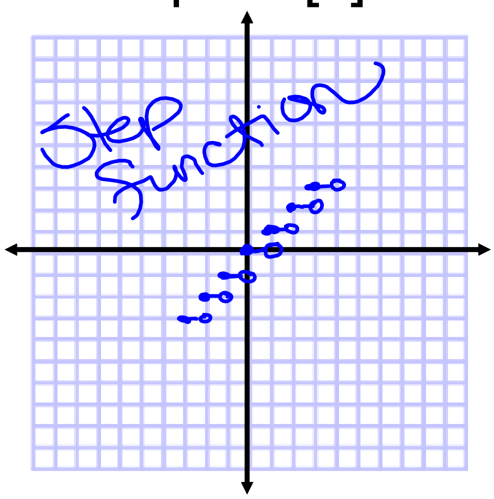
[x]

[1.5] = 1 [-2.5] = -3

[2.88] = 2 [-1.222] = -2




Graph: [x]



# Properties of Limits

10/2

$$\lim_{x \to a} f(x) = L \quad \text{and} \quad \lim_{x \to a} g(x) = M$$

(a) $\lim_{x \to a} \{f(x) + g(x)\} = \qquad =$

(b) $\lim_{x \to a} \{f(x) - g(x)\} = \qquad =$

(c) $\lim_{x \to a} k\, f(x) = \qquad =$

(d) $\lim_{x \to a} f(x) \cdot g(x) = \qquad =$

(e) $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \qquad =$ (if $M \neq 0$

(f) $\lim_{x \to a} \{ f(x) \}^n = \qquad =$

(g) $\lim_{x \to a} \sqrt[n]{f(x)} = \qquad =$

(if $L > 0$ when $n$ is even)

$$f(x) = x^2 - x - 6 \text{ and } g(x) = x^2 - 2x - 3$$

(a) $\lim_{x \to 1} \{f(x) + g(x)\}$

$\lim_{x \to 1} (x^2 - x - 6) + \lim_{x \to 1} (x^2 - 2x - 3)$

$-6 + -4$

$-10$

(b) $\lim_{x \to 1} f(x)g(x)$

$-6 \cdot -4$

$24$

(c) $\lim_{x \to 1} f(x)/g(x)$

$\frac{3}{2}$

(d) $\lim_{x \to 3} \{f(x) + g(x)\}$

$0$

(e) $\lim_{x \to 3} f(x)g(x)$

$0$

(f) $\lim_{x \to 3} f(x)/g(x)$

DNE

(g) $\lim_{x \to 2} \{ f(x) \}^3$

(h) $\lim_{x \to 2} \sqrt{1 - g(x)}$

## The limit of composite functions

$$\lim_{x\to c} f(g(x)) = \quad = $$

$$\lim_{x\to 0}\sqrt{x^2+4}$$

$$\sqrt{\lim_{x\to 0} x^2+4}$$

$$\sqrt{4}$$

$$2$$

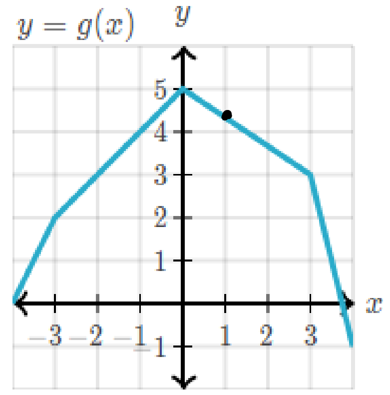
$$\lim_{x \to 0} \sqrt[3]{2x^2 - 10}$$

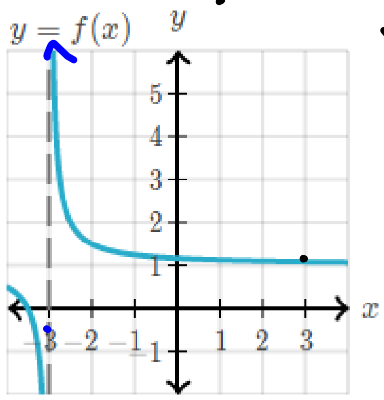
$= \sqrt[3]{-10}$

$-\sqrt[3]{10}$

$$\lim_{x \to -3} g(f(x))$$

$\lim_{x \to -3} f(x)$ DNE




$\lim_{x \to 3} g(f(x))$

$\lim_{x \to 3} f(x)$

$g(1) = 4.3$

# Limits of Trig Functions

Let $c$ be a real number in the domain of the given trigonometric function.

1. $\lim_{x \to c} \sin x = \sin c$
2. $\lim_{x \to c} \cos x = \cos c$
3. $\lim_{x \to c} \tan x = \tan c$
4. $\lim_{x \to c} \cot x = \cot c$
5. $\lim_{x \to c} \sec x = \sec c$
6. $\lim_{x \to c} \csc x = \csc c$

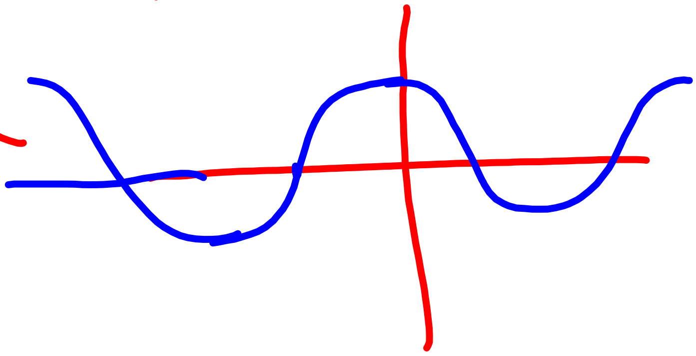
$$\lim_{x\to\pi}(x\cos x)$$

$$\lim_{x\to\pi} x \cdot \lim_{x\to\pi} \cos x$$

$$\pi \cdot \cos\pi$$

$$\pi \cdot -1$$

$$-\pi$$

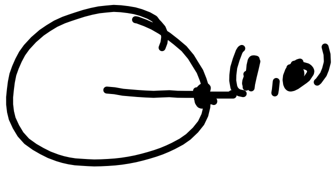


$$\lim_{x\to 0} \sin^2 x$$

$$\left(\lim_{x\to 0} \sin x\right)^2$$

$$(0)^2$$

$$0$$



# Rationalizing Technique

$$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$$

1. Try direct substitution.

0/0 Indeterminant form

$$\frac{\sqrt{x+1} - 1}{x} \cdot \frac{(\sqrt{x+1} + 1)}{(\sqrt{x+1} + 1)}$$

Rationalize the numerator

$$\frac{x + \cancel{1} + \cancel{\sqrt{x+1}} - \cancel{\sqrt{x+1}} - \cancel{1}}{x(\sqrt{x+1} + 1)} = \frac{\cancel{x}\ 1}{\cancel{x}(\sqrt{x+1} + 1)}$$

$$\lim_{x \to 0} \frac{1}{\sqrt{x+1} + 1} = \frac{1}{\sqrt{0+1} + 1} = \frac{1}{2}$$